THERMAL FIELDS IN REGIONS OF COMPLEX CONFIGURATION

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On the basis of probabilistic concepts, a simple method of weighted averaging of the boundary temperatures is developed for the construction of the steady thermal field in regions of complex geometry.

The problem of constructing a steady temperature field U(M) within a region Ω at specified temperature at the boundary ∂Ω of this region reduces to solving the Laplace equation

$$\Delta U = 0 \quad (1)$$

with the boundary condition of the first kind (Dirichlet condition)

$$U|_{\partial\Omega} = \varphi(M). \quad (2)$$

This typical problem of the derivation of a function is very important in physical and technical applications. It may be interpreted as a problem of finding the electrostatic potential within a region from the known potential distribution at the boundary. Another interpretation is as a model of a soap film. The urgent problem of determining the productivity of an oil field also reduces to the solution of a Dirichlet problem [1]. Analogous problems arise in metrology, topography, in the interpretation of various geophysical experimental data, and elsewhere.

For geometrically trivial regions, it is simple to construct an accurate solution of the Dirichlet problem. However, in practically important cases, regions of complex configuration must be dealt with. As a rule, the difficulties arising here may be successfully overcome by numerical methods: the finite-difference method (FDM) with an adaptive computational template for working at complex boundaries; or the finite-element method (FEM), which works well in the solution of difficult engineering problems.

It is supposed that the simplest algorithm solving problem (1), (2) will be given by the Monte carlo method [2], based on the principles of random motion between the points of an integer lattice. In fact, the computational procedure of the Monte Carlo method is very simple, but randomization of the calculation requires a sufficiently powerful computer fitted with a special device for generating random codes.

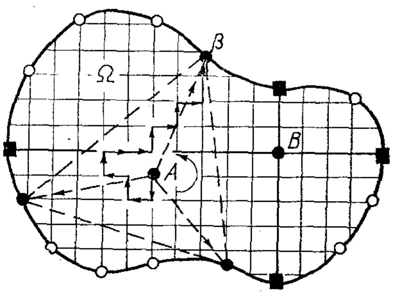

Fig. 1

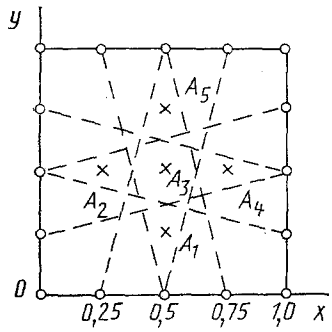

Fig. 2

Fig. 1. Zigzag path of Monte Carlo method; stop-frame method of simplex rotation; adaptive FDM template.

Fig. 2. Calculation points A_i at a plate and four SRM stop frames.

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TABLE 1. Temperature Values (°C) at Calculation Points of Plate

Calctn. points	Accurate solution	Solution by	
		SRM	AT
A_1	46,879	48,944	48,041
A_2	21,094	21,787	21,612
A_3	44,531	45,656	44,782
A_4	71,094	71,796	71,481
A_5	38,058	38,742	38,658

The Dirichlet problem is a model problem for the development of new methods and the testing of new ideas, which may then be extended to the general theory of partial differential equations. In the present work, a new method combining the fruitful ideas of the Monte Carlo method and the simplest finite-element approximations is used to solve the Dirichlet problem in regions of complex configuration.

In the standard Monte Carlo method, the temperature at point A of region Ω (Fig. 1) is determined by the formula

$$U(A)=\frac{1}{m}\sum_{\beta=1}^{N} m_\beta U_\beta, \tag{3}$$

where N is the number of points at the boundary of the region; m is the total number of paths of the particle starting from point A; m_β is the number of a particle absorption at point β. The well-known Bernoulli theorem [3] guarantees the convergence (in terms of the probability) of a sequence of relative frequencies m_β/m to some limit Φ_β, which, as shown in [4], is the geometric probability. It was also established in [4] that the probabilities of absorption of the traveling particle at points of a simplex element coincide precisely with the baricentric coordinates of the simplex. This means that, in solving the Dirichlet problem, attention may be confined to a single simplex element (Fig. 1), offering the possibility of rotating it so that new boundary points are systematically included in the calculation. To determine the temperature at point A, it is sufficiently accurate to use several "stop frames" fixing various positions of the simplex and then to average the results obtained. In each stop frame in the plane problem, the following formula is used instead of Eq. (3)

$$U(A)=\sum_{\beta=1}^{3}\Phi_\beta U_\beta. \tag{4}$$

In three-dimensional problems, this formula includes four terms, since the tetrahedron is rotated in the region Ω.

The simplex-rotation method (SRM) is limitingly simple and suitable for regions of very general form; it has rapid convergence and, in contrast to the Monte Carlo method, may easily be realized on a microcalculator. An elementary a posteriori estimate of the SRM error was obtained in [4] on the basis of dispersion of the target function.

As an example, consider the problem [5] of a steady temperature distribution in a square plate, at the sides x = 0 and x = 1 of which temperatures of 0 and 100°C, respectively, are maintained; at the side y = 0, the temperature increases linearly, while at y = 1 it increases according to a quadratic-parabola law. The Dirichlet problem was solved in [5] by the grid method with 25 points (9 internal points). The accurate solution obtained by a direct method is obtained after 30 iterations of the simultaneous-displacement method, 16 iterations of the successive-displacement method, and nine iterations of the method of successive upper relaxation. Practically the same accuracy is obtained by SRM using no more than four stop frames (Fig. 2). The calculation results are given in Table 1, which gives an idea of the SRM accuracy.

Probabilistic models significantly change our concepts regarding the nature of the computational algorithms, offering new possibilities for further simplification. For example, the orthogonal computational template of "cross" type used in the standard grid method may be adapted for the solution of the Dirichlet problem in regions of complex geometry. It is sufficient here to regulate the length of the template arm close to the curvilinear boundary. This possibility is ensured by simple change in the rules of random particle motion over orthogonal trajectories. In Fig. 1, this adaptive FDM template is "tied" to point B. If the adaptive template (AT) is also systematically rotated around the point B, the contributions of other boundary points may be taken into account. As in SRM, several stop frames are used to achieve acceptable accuracy, with subsequent averaging of the results. Table 1 gives the results of calculations using AT for four stop frames. Comparison of the two simplified approaches shows that the additional path in the random-motion scheme enriches the information at the given point and, with the same number of stop frames, increases the calculation accuracy, as a rule.

NOTATION

$U(M)$, temperature at point M of region Ω; $\partial\Omega$, boundary of region; U_β, temperature at boundary point β; m_β/m, relative frequency of absorption of moving particle at point β; Φ_β, baricentric coordinates of point A in simplex.

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THERMAL HYSTERESIS IN NONLINEAR MEDIA

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One-dimensional thermal relaxation processes in media with nonlinear thermophysical properties are treated. Dynamic hysteresis is investigated theoretically in continuous and discontinuous nonstationary temperature fields. Boundary conditions are analyzed, for which a high-flow hysteresis process is realized. A quantitative estimate is given of irreversible variations in the material thermal state. Examples are given of constructing hysteresis branches. New properties are established for thermal shock waves propagating along the relaxing background.

It is well-known that hysteresis effects are observed in various physical processes (magnetic hysteresis [1], elastic hysteresis [2], etc.), and are characterized by a nonunique dependence between the quantities determining the material state and the external conditions of action. As applied to heat and mass transfer, these effects were noted in [3-6]. The mathematical methods of analyzing systems with nonlinear hysteresis were discussed in [7].

The purpose of the present study is construction of examples of analytic description of dynamic thermal hysteresis, realized during fast flow processes, both in continuous and discontinuous thermal fields. The mathematical model is based on the energy equation and on the generalized Fourier law [8, 9], written in dimensionless form

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